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DESIGNS FOR INDUSTRIAL EXPERIMENTATION

PART 1 STANDARD DESIGNS

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NEW DELHI 110002

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Indian Standard

DESIGNS FOR INDUSTRIAL EXPERIMENTATION

PART 1 STANDARD DESIGNS

Quality Control and Industrial Statistics Sectional Committee, EC 3

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Indian Standard

DESIGNS FOR INDUSTRIAL EXPERIMENTATION

PART 1 STANDARD DESIGNS

0. FOREWORD

0.1 This Indian Standard was adopted by the Indian Standards Institution on 15 December 1982, after the draft finalized by the Quality Control and Industrial Statistics Sectional Committee had been approved by the Executive Committee.

0.2 Industrial organizations are constantly faced with the problem of decision making regarding product/process design, process specifications, quality improvement, dominant factors affecting quality, cost reduction, import substitution, etc. In all such problems, one is confronted with several alternatives and one has to choose that alternative which satisfies the requirements at minimum cost. For taking a right decision in all such cases, an experiment may have to be carried out either to discover something about a particular process or to compare the effect of several conditions on the phenomenon under study.

0.3 The effectiveness of an experiment depends to a large extent on the manner in which the data is collected. The method of data collection may adversely affect the conclusions that can be drawn from the experiment. If, therefore, proper designing of an experiment is not made, it may happen that no inferences may be drawn or if drawn, may not answer the questions to which the experimenter is seeking an answer. The designing of an experiment is essentially the determination of the pattern of observations to be collected. A good experimental design is one that answers efficiently and unambiguously those questions which are to be resolved and furnishes the required information with a minimum of experimental effort. For this purpose the experiments may be statistically designed.

0.4 The main advantage of designing an experiment statistically is to obtain unambiguous results at a minimum cost. The statistically designed experiments also enable the experimenter to :

a) isolate the effects of known extraneous factors,
b) evaluate the inter-relationship or interaction between the factors,
c) evaluate the experimental error,
d) determine beforehand the size of the experiment for the specified precision in the results,
e) extract maximum information from the data, and
f) remove uncertainty from the conclusions.

0.5 This edition 1.1 incorporates Amendment No. 1 (February 1992). Side bar indicates modification of the text as the result of incorporation of the amendment.

0.6 In reporting the results of tests or analyses, if the final value, observed or calculated, is to be rounded off, it shall be done in accordance with IS : 2-1960*.

1. SCOPE

1.1 This standard provides methods of planning and conducting experiments under various conditions. It also describes the procedures for analysing data recorded from such experiments. The various designs described in the standard are completely randomised design, randomised block design, latin square design, balanced incomplete block design and factorial designs.

2. TERMINOLOGY

2.0 For the purpose of this standard the following definitions shall apply.

2.1 Experimental Design — The arrangement in which an experimental programme is to be conducted and the selection of the levels of one or more factors or factor combinations to be included in the experiment.

2.2 Randomisation — The procedure of allocating treatments at random to experimental units.

2.3 Replication — The repetition of treatments over experimental units under similar conditions.

2.4 Experimental Error — The experimental error is that part of the total variation which cannot be assigned to any given cause or which is not associated with any deliberate variation in the experimental conditions.

2.5 Blocks — A set of homogeneous experimental units which give rise to observations, among which the error variation is expected to be less than the whole set of observations.

2.6 Degrees of Freedom — The number of independent component values which are necessary to determine a statistic.

3. COMPLETELY RANDOMISED DESIGN

3.1 This is the simplest pattern of collecting experimental data when the experimenter is interested in testing the effect of a set of treatments. In this design the experimental units are allotted at random to the treatments, so that every experimental material unit

*Rules for rounding off numerical values (*revised*).

gets the same chance of receiving every treatment. In addition the units should be processed in random order at all subsequent stages in the experiment where this order is likely to affect the results. Let the total experiment material consist of *n* homogeneous experimental units and there be a set of *t* treatments to be tested. The treatments are then allocated randomly to all the experimental units. For this purpose a table of random numbers may be used to assign the units to the treatments. For instance, let the *i*th treatment be replicted r_i times so that $\sum_{i=1}^{t} r_i = n$. The treatments may be numbered arbitrarily from *1* to *t* and the experimental units from *1* to *n*. r_i units selected at random from the *n* units, using a table of random numbers, may be allotted to the first treatment, r_2 units selected randomly from the remaining units to the second treatment, and so on.

3.2 The observations obtained from the *n* experimental units are to be recorded as shown in Table 1.

TABLE 1 DISTRIBUTION OF TREATMENTS IN EXPERIMENTAL UNITS

	Treatments				
	1	2	3		t
	y_{11}	y_{21}	y_{31}	....	y_{t_1}
	y_{12}	$y_{\_2}$	y_{32}	....	y_{t_2}
	y_{13}	y_{23}	y_{33}	....	y_{t_3}
	y_{1r_1}	y_{2r_2}	y_{3r_3}	....	y_{tr_t}
Total	T_1	T_2	T_3	....	T_t

Grand total = $T_1 + T_2 + ... + T_t = G$

3.3 From Table 1, the following sums of squares are calculated :

a) Correction factor for mean (CF) :
Square the grand total G and divide by *n*

$$CF = \frac{G^2}{n}$$

b) Total sum of squares (S_{tot}) :
Square each y_{ij}-value and add. From this sum subtract CF,

$$S_{tot} = \sum_i \sum_j y_{ij}^2 - CF$$

c) Sum of squares between treatments (S_T) :
Square the treatment totals (T_i) and divide them by the number of observations (r_i) on which each total is based and add. Subtract from this CF.

$$S_T = \frac{T_1^2}{r_1} + \frac{T_2^2}{r_2} + \frac{T_3^2}{r_3} + \ldots + \frac{T_i^2}{r_t} - CF = \sum_{i=1}^{t} \frac{T_i^2}{r_i} - CF$$

d) Sum of squares due to experimental error (S_E) :

$$S_E = S_{tot} - S_T = \sum_i \sum_j y_{ij}^2 - \sum_{i=1}^{t} \frac{T_i^2}{r_i}$$

3.4 The different sum of squares can be entered in the analysis of variance table as shown in Table 2.

TABLE 2 ANALYSIS OF VARIANCE TABLE FOR COMPLETELY RANDOMISED DESIGN

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F-value
(1)	(2)	(3)	(4)	(5)
Between treatments	(*t-1*)	$S_T = \sum_i \frac{T_i^2}{r_i} - CF$	$MS_T = \frac{S_T}{(t-1)}$	$\frac{MS_T}{MS_E}$
Experimental error	(*n-t*)	$S_E = \sum_i \sum_j y_{ij}^2 \sum_i \frac{T_i^2}{r_i}$	$MS_E = \frac{S_E}{(n-t)}$	
Total	(*n-1*)	$S_{tot} = \sum_i \sum_j y_{ij}^2 - CF$		

3.5 The mean square column in Table 2 is obtained by dividing sum of square component by its degrees of freedom.

3.6 The interest of the experimenter lies in testing the null hypothesis that all treatments effects are equal. For this, the statistic $F = MS_T/MS_E$ is computed. This value of F is then compared with the tabulated F-value (variance ratio table) at desired level of significance (say 5 percent) for [(*t-1*), (*n-t*)] degrees of freedom. If the calculated value of F exceeds the tabulated value, the null hypothesis is rejected, meaning thereby that there is sufficient reason to believe that all treatment effects are not equal.

3.7 If the null hypothesis of all treatment effects being equal is rejected, proceed to calculate the critical difference between two treatments. The critical difference between any two treatments, say t_i and t_j is given as follows :

$$\text{Critical difference (CD)} = \left[\left(\frac{1}{r_i} + \frac{1}{r_j} \right) \times MS_E \right]^{\frac{1}{2}} \times t\alpha$$

where $t\alpha$ is the α% value of t-variate with (n-t) degrees of freedom, α being the level of significance.

3.7.1 The mean for each treatment is then calculated as $\bar{y}_i = T_i/r_i$.

3.7.2 The absolute difference between any two treatment means is compared with its critical difference. If the difference between the two treatment means is less than the critical difference, the two treatments are not considered to be significantly different. If the absolute difference between two treatment means is greater than the critical difference, it is considered that these are significantly different.

3.8 Merits and Demerits

3.8.1 *Merits*

a) It allows complete flexibility as any number of treatments and replicates may be used. The number of replicates, if desired, can be varied from treatment to treatment;

b) The statistical analysis is easy even if the number of replicates are not the same for all treatments or if the experimental errors differ from treatment to treatment; and

c) The relative loss of information due to missing observations is smaller than with any other design and they do not pose any problem in the analysis of data.

3.8.2 *Demerits* — As the experimental units are heterogeneous, the units that receive one treatment may not be similar to those which receive another treatment and so the entire variation among the units enters into the experimental error. So the experimental error is higher as compared to other designs.

3.9 Complete randomisation is therefore only appropriate, where

a) the experimental units are homogeneous, and

b) an appreciable fraction of the units is likely to be destroyed or fail to respond.

3.10 Example 1 — Six hourly samples were taken from each of the three different types of mixes during grinding of final cement for determination of rapid calcium oxide content. The test results of the samples are given in Table 3. It has to be tested whether the different types of mixes produce the same calcium oxide content.

TABLE 3 CALCIUM OXIDE CONTENT (IN PERCENT)

(*Clause* 3.10)

	MIX I	MIX II	MIX III
	(1)	(2)	(3)
	43.9	45.0	44.2
	44.0	45.1	44.4
	43.9	45.1	44.1
	44.0	45.2	44.0
	43.9	45.0	44.2
	44.1	45.0	44.1
Total	263.8	270.4	265.0, G = 799.2

3.10.1 The various calculations are as given below:

a) Correction factor (CF) = $\frac{(799.2)^2}{18}$ = 35 484.48

b) Total sum of squares

$= [(43.9)^2 + (44.0)^2 + \ ...... \ + (44.1)^2] - CF$

$= 4.28$

c) Sum of squares between mixes

$= \frac{(263.8)^2}{6} + \frac{(270.4)^2}{6} + \frac{(265.0)^2}{6} - CF = 4.12$

d) Sum of squares due to error = 4.28 – 4.12 = 0.16

3.10.2 The analysis of variance table for above experiment is given in Table 4.

TABLE 4 ANALYSIS OF VARIANCE TABLE

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SQUARE	F-VALUE
(1)	(2)	(3)	(4)	(5)
Between mixes	2	4.12	2.06	187
Experimental error	15	0.16	0.011	
Total	17	4.28		

3.10.3 The tabulated value of F at 5 percent level of significance, for (2,15) degrees of freedom is 3.68, which is less than the calculated value. Hence the three different types of mixes are significantly different, as far as calcium oxide content is concerned.

3.10.4 The critical difference for different pairs is calculated next. For instance, the critical difference value at 5 percent level, for mixes I and III is given by:

$$\text{Critical difference (CD)} = \left[\left(\frac{1}{6}+\frac{1}{6}\right)\times 0.011\right]^{\frac{1}{2}} \times 2.131$$

$$= 0.13$$

3.10.4.1 The average for mix I ($\bar{y}_1$) = 43.97 and for mix III ($\bar{y}_3$) = 44.17. Since $|\bar{y}_1 - \bar{y}_3|$ (= 0.20) is greater than the critical difference value, mixes I and III are significantly different at 5 percent significance level.

4. RANDOMISED BLOCK DESIGN

4.1 In this case, the total experimental material is divided into 6 blocks or groups of homogeneous material and then, each block is sub-divided into *t* experimental units, where *t* denotes the number of treatments to be tested.

4.2 The *t* treatments are then allocated to *t* experimental units within each block at random. Separate random allocation is made for each block. For random allocation, first *t* treatments are given random numbers from *1* to *t*. Then, a random number between *1* and *t* is chosen and whatever number comes, the treatment having that number is allocated to the first experimental unit in a block. The procedure is continued till all the treatments are allocated within a block. The procedure is then repeated with the other blocks.

4.3 The observations obtained from the *bt* experimental units are to be recorded as given in Table 5.

TABLE 5 DISTRIBUTION OF TREATMENTS IN BLOCKS

Treatment \ Block	1	2	3	....	b	Total
1	y_{11}	y_{12}	y_{13}	....	y_{1b}	T_1
2	y_{21}	y_{22}	y_{23}	....	y_{2b}	T_2
.						
.						
.						
t	y_{t1}	y_{t2}	y_{t3}	....	y_{tb}	T_t
Total	B_1	B_2	B_3	....	B_b	G

4.4 From the above table, the following sums of squares are calculated:

a) Correction factor (CF) = $\frac{G^2}{bt}$

b) Total sum of squares (S_{tot}) = $(y_{11}^2 + y_{12}^2 + \ldots\ldots + y_{tb}^2) - CF$

$$= \sum_i \sum_j y_{ij}^2 - CF$$

c) Sum of squares between treatments (S_T)

$$= \frac{1}{b}(T_1^2 + T_2^2 + \ldots\ldots + T_t^2) - CF$$

$$= \frac{1}{b} \sum_{i=1}^{t} T^2{}_i - CF$$

d) Sum of squares between blocks (S_B)

$$= \frac{1}{t}(B_1^2 + B_2^2 + \ldots\ldots + B_b^2) - CF = \sum_{j=1}^{b} \frac{B_j^2}{t} - CF$$

e) Sum of squares due to experimental error (S_E) = $S_{tot} - S_T - S_B$

4.5 The different sums of squares may now be entered in the analysis of variance table as shown in Table 6.

TABLE 6 ANALYSIS OF VARIANCE TABLE FOR RANDOMISED BLOCK DESIGN

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SQUARE	F-VALUE
(1)	(2)	(3)	(4)	(5)
Between blocks	*b-1*	$S_B = \frac{1}{t} \sum_j B_j^2 - CF$	$MS_B = S_B/(b-1)$	$\frac{MS_B}{MS_E}$
Between treatments	*t-1*	$S_T = \frac{1}{b} \sum_i T_i^2 - CF$	$MS_T = S_T/(t-1)$	$\frac{MS_T}{MS_E}$
Experimental error	(*b-1*) (*t-1*)	$S_E = S_{tot} - S_B - S_T$	$MS_E = S_E/(b-1)(t-1)$	
Total	*bt – 1*	$S_{tot} = \sum_i \sum_j y_{jj}^2 - CF$		

4.6 Since the interest of the experimental lies in testing the hypothesis whether all treatment effects are identical, the following statistic is computed :

$$F = MS_T/MS_E$$

4.6.1 The calculated value of F is then compared with the tabulated F-value (variance ratio table) at desired level of significance (say 5 percent) for [(*t-1*), (*b-1*) (*t-1*)] degrees of freedom. If the calculated value of F is less than the tabulated value, the null hypothesis is not rejected meaning thereby that there is no significant difference among treatments. If the calculated value of F exceeds the tabulated value, reject the null hypothesis meaning thereby that there is sufficient reason to believe that all treatment effects are not equal.

4.7 Merits and Demerits

4.7.1 *Merits*

a) By means of grouping, the experimental error is reduced and the treatment comparisons are made more sensitive than with completely randomised designs;

b) Any number of treatments and replicates may be included. However, if the number of treatments is large (20 or more), the efficiency of error control decreases; and

c) The statistical analysis is straightforward. When data from some individual units are missing, the 'missing plot' technique enables the available results to be fully utilized.

4.7.2 *Demerits*

a) The effect of only one factor is investigated, and

b) Single grouping of only one factor yields higher experimental error as compared to more complicated designs.

4.8 Example — An experiment was conducted for ash content of instant tea. For this purpose five different varieties of instant tea were tested in three laboratories. The test results are given in Table 7. It has to be determined whether there is significant difference among varieties of instant tea.

4.8.1 The various calculations are obtained as follows:

a) Correction factor (CF) = $\dfrac{(299.8)^2}{15} = 5\,992.00$

b) Total sum of squares =

$$[(20.1)^2 + (20.1)^2 + \ldots + (20.5)^2] - CF = 9.86$$

TABLE 7 ASH CONTENT OF INSTANT TEA

(*Clause* 4.8)

VARIETY \ LABORATORY	A	B	C	TOTAL
1	20.1	20.1	20.3	60.5
2	20.5	20.3	20.2	61.0
3	20.2	20.0	20.0	60.2
4	18.7	18.3	18.5	55.5
5	21.2	20.9	20.5	62.6
Total	100.7	99.6	99.5	299.8

c) Sum of squares between varieties of tea

$$= \frac{1}{3} [(60.5)^2 + (61.0)^2 + ... + (62.6)^2] - CF$$

$= 9.43$

d) Sum of squares between laboratories

$$= \frac{1}{5} [(100.7)^2 + (99.6)^2 + (99.5)^2] - CF$$

$= 0.18$

e) Sum of squares due to experimental error

$= 9.86 - 9.43 - 0.18 = 0.25$

4.8.2 The analysis of variance table for the above data is given in Table 8.

TABLE 8 ANALYSIS OF VARIANCE TABLE

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SQUARE	F-VALUE
(1)	(2)	(3)	(4)	(5)
Between varieties	4	9.43	2.36	78.7
Between laboratories	2	0.18	0.09	3.0
Experimental error	8	0.25	0.03	
Total	14	9.86		

4.8.3 The tabulated value of F for (4, 8) degrees of freedom at 5 percent significance level is 3.84, which is less than the calculated F-value of 78.7. So it can be concluded that there is significant difference among the varieties of instant tea.

5. LATIN SQUARE DESIGN

5.1 In the latin square design, the treatments are classified in complete groups in two directions, the two classifications being orthogonal to each other and also to the treatments. Each row and each column is a complete replication. The effect of the double grouping is to eliminate from the errors, all differences among rows, as also all the differences among columns. The experimental material should be arranged and the experiment conducted so that the differences among rows and columns represent major sources of variation.

5.2 For four treatments A, B, C and D, the latin square arrangement lay out may be as follows:

Row \ Column	1	2	3	4
1	A	B	C	D
2	D	A	B	C
3	C	D	A	B
4	B	C	D	A

5.2.1 Each row and each column contains a complete set of treatments. Such a latin square is said to be a latin square of side 4. In general, if we have a latin square of side r, then each of the r treatments is replicated r times and each treatment occurs once and only once in every row and every column.

5.3 A latin square is said to be standard if the first row and first column are in the standard order. The standard latin squares of side 4, 5 and 6 are given in Appendix A. There are two standard latin squares of side 4, three standard latin squares of side 5 and 22 standard latin squares of side 6. The key numbers are given below each standard latin square. The procedure for selection of a latin square at random is illustrated in **5.3.1** and **5.3.2**.

5.3.1 If 5 × 5 latin square is required, it can be seen by referring to Appendix A, that the largest key number for 5 × 5 latin square is 56. Hence a random number from 1 to 56 is chosen. Let the random number chosen be 18. Since 18 is in the range of key numbers 1 to 25, standard latin square number 1 is selected. Next, all the rows of the selected standard latin square are permuted at random by selecting the digits from 1 to 5 from the random number tables given in IS : 4905-1968*. The procedure of selection of digits at random is given in **4.2** of IS : 4905-1968*. Let, for example, the order of digits obtained at random be 4, 3, 1, 5 and 2. This implies that fourth row shall be the first row, the third row shall come next and so on. After permuting the rows and random, all the columns are permuted at random in the same way. The letters to the treatments are then assigned at random.

5.3.2 For latin squares of sides 7 to 10, one square for each side is given in Appendix A, from which any square of the transformation sets may be generated by the random permutation of all rows and columns. The letters to the treatments are then assigned at random. The random permutation may be done as in **5.3.1**.

5.4 The observations from t^2 experimental units (for 4 treatments) can be recorded as given in Table 9.

TABLE 9 DISTRIBUTION OF TREATMEMTNS IN t^2 EXPERIMENTAL UNITS

Column / Row	1	2	3	4	Total
1	A (y_{11})	B (y_{12})	D (y_{13})	C (y_{14})	R_1
2	D (y_{21})	C (y_{22})	A (y_{23})	B (y_{24})	R_2
3	C (y_{21})	D (y_{32})	B (y_{33})	A (y_{34})	R_3
4	B (y_{41})	A (y_{42})	C (y_{43})	D (y_{44})	R_4
Total	C_1	C_2	C_3	C_4	G

*Methods for random sampling.

5.5 From the above table, the following sums of squares are calculated:

a) Correction factor (CF) = $\frac{G^2}{t^2}$, where G is grand total.

b) Total sum of squares (S_{tot}) = $\sum_i \sum_j y_{ij}^2 - CF$

c) Sum of squares between treatments (S_T) :
Collect the observations for each treatment and get *t* treatment totals, say T_1, T_2......... etc.

$$S_T = \frac{1}{t} \sum_{i=1}^{t} T_i^2 - CF$$

d) Sum of squares between rows (S_R) = $\frac{1}{t} \sum_{i=1}^{t} R_i^2 - CF$

e) Sum of squares between columns (S_C) = $\frac{1}{t} \sum_{i=1}^{t} C_i^2 - CF$

f) Sum of squares due to experimental error (S_E)
$= S_{tot} - S_R - S_C - S_T$

5.6 The different sums of squares can now be entered in the following analysis of variance table:

TABLE 10 ANALYSIS OF VARIANCE TABLE FOR LATIN SQUARE DESIGN

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F-value
(1)	(2)	(3)	(4)	(5)
Between rows	*t-1*	$S_R = \frac{1}{t} \sum_i R_i^2 - CF$	$MS_R = S_R/(t-1)$	$\frac{MS_R}{MS_E}$
Between columns	*t-1*	$S_C = \frac{1}{t} \sum_i C_i^2 - CF$	$MS_C = S_C/(t-1)$	$\frac{MS_C}{MS_E}$
Between treatments	*t-1*	$S_T = \frac{1}{t} \sum_i T_i^2 - CF$	$MS_T = S_T/(t-1)$	$\frac{MS_T}{MS_E}$
Experimental error	(*t*-1) (*t*-2)	$S_E = S_{tot} - S_R - S_C - S_T$	$MS_E = \frac{S_E}{(t-1)(t-2)}$	
Total	t^2 - 1	$\sum_i \sum_j y_{ij}^2 - CF$		

5.7 For testing the differences between treatments or rows or columns, the mean square for that particular component is compared with the mean square due to experimental error, that is,

$$F_1 = MS_T/MS_E,\ F_2 = MS_R/MS_E \text{ and } F_3 = MS_C/MS_E$$

5.7.1 If the computed value of F_1, (say) exceeds the tabulated value of F at desired level of significance for [(*t*-1), (*t*-1) (*t*-2)] degrees of freedom, the hypothesis that all treatment effects are equal is rejected. The next step is to calculate the critical differences in a similar fashion as given in **4.7**.

5.8 Merits and Demerits

5.8.1 *Merits*

a) More than one factor can be investigated simultaneously and with fewer replications than more complicated designs, and

b) Due to double grouping, latin square provides more opportunity than randomised block design for the reduction of errors.

5.8.2 *Demerits*

a) The design has a restriction that each treatment may occur only once in each row and each column,

b) An assumption is made that the factors are independent, and

c) Latin square is not suitable when the number of treatments is large as there will be as many replications as there are treatments. Moreover, when there are more treatments, it may be difficult to allocate the rows and the columns to sources of variability in an efficient manner.

5.9 Example 3 — In order to study the effect of relative humidity (RH) on the warp-way tensile strength of a standard hessian fabric, one 3 × 1.5 m fabric sample was cut from a 100 × 1.5 m hessian of the above type. Twenty-five fabric test specimens each of dimension 10 × 20 cm were then cut from five different rows of the fabric sample, each row containing five test specimen. In each row, five specimen were tested under five different relative humidities. So at each relative humidity, five strength tests were carried out. The plan of selecting the test specimens and the test results (kg) obtained are as given in Table 11.

5.9.1 In Table 11, each cell may be considered as a test specimen of dimension 10 × 20 cm. The alphabet within a cell represents the humidity at which the test specimen is tested, while the numerical value appearing at the bottom within the same cell represents the strength value (kg) obtained at the particular humidity.

TABLE 11 TENSILE STRENGTH VALUES (kg) OF FABRIC

(*Clause* 5.9)

Row \ Column	1	2	3	4	5	Total
1	A 90	B 105	C 115	D 120	E 122	552
2	B 108	C 117	D 125	E 123	A 95	568
3	C 112	D 124	E 120	A 93	B 100	549
4	D 119	E 118	A 94	B 102	C 110	543
5	E 121	A 98	B 106	C 114	D 126	565
Total	550	562	560	552	553	2 777

5.9.2 Variation between rows components in this case represents the variation in the warp-way strength along the different portion of the same warp thread, the variation between columns represents the variation in the strength between different warp threads and the different humidities stand for different treatments.

5.9.3 Treatment totals are as given below:

$T_A = 470$, $T_B = 521$, $T_C = 568$, $T_D = 614$ and $T_E = 604$

5.9.4 The various calculations are as given below:

a) Correction factor (CF) = $\frac{(2\ 777)^2}{25} = 308\ 469.2$

b) Total sum of squares = $[\ (90)^2 + (105)^2 + \ldots + (126)^2\ ] - CF$
$= 3\ 043.8$

c) Sum of squares between rows:

$\frac{1}{5}[\ (552)^2 + (568)^2 + (549)^2 + (543)^2 + (565)^2\ ] - CF$

$= 91.4$

d) Sum of squares between columns :

$= \frac{1}{5}[(550)^2 + (562)^2 + (560)^2 + (552)^2 + (553)^2] - CF$

$= 22.2$

e) Sum of squares between treatments:

$= \frac{1}{5}[(470)^2 + (521)^2 + (568)^2 + (614)^2 + (604)^2] - CF$

$= 2\ 886.2$

f) Sum of squares due to experimental error

$= 3\ 043.8 - 91.4 - 22.2 - 2\ 886.2$
$= 44.0$

5.9.5 The analysis of variance table is given in Table 12.

TABLE 12 ANALYSIS OF VARIANCE TABLE

Sources of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F-value
(1)	(2)	(3)	(4)	(5)
Between rows	4	91.4	22.8	6.16
Between columns	4	22.2	5.6	1.51
Between treatments	4	2 886.2	721.5	195
Experimental Error	12	44.0	3.7	
Total	24	3 043.8		

5.9.6 The calculated value of F for treatments is 195, whereas the tabulated value of F for (4, 12) degrees of freedom at 5 percent and 1 percent levels are 3.26 and 5.41 respectively. Thus the variation among treatments is highly significant at both these levels. It may be seen that the variation between rows is also significant at both these levels.

6. BALANCED INCOMPLETE BLOCK DESIGN

6.1 Experiments for comparing a number of treatments when uniform conditions can be maintained within blocks can be planned by using a randomised block design. However, in certain situations the experimental conditions do not permit blocks large enough to include every treatment, so that not all treatments are included in every block. In a situation of this type, designs known as incomplete block designs can be used. Balanced incomplete blocks are a class of important incomplete block designs.

6.2 In a balanced incomplete block (BIB) design, the experimental material is divided into 'b' blocks of 'k' units each ($k < t$), where t is the total number of treatments to be tested. Each treatment is replicated 'r' times, that is, the treatments are so arranged that each treatment occurs in exactly 'r' blocks and further, each pair of treatment occurs together in 'λ' blocks. The integers t, b, r, k and λ are called the parameters of the design. The constraints on the parameters of the design are as follows:

a) $t > k$,

b) $tr = bk$, and

c) $\lambda (t-1) = r(k-1)$

6.3 The following is a lay-out of a BIB design, with 6 treatments in blocks of size 3. The numbers in the brackets are the treatment numbers:

Blocks	*Treatments*
1	(1, 2, 5)
2	(6, 2, 1)
3	(3, 1, 4)
4	(4, 3, 2)
5	(3, 5, 6)
6	(1, 3, 6)
7	(5, 3, 2)
8	(4, 5, 6)
9	(6, 4, 2)
10	(4, 1, 5)

Here $t = 6$, $b = 10$, $r = 5$, $k = 3$, and $\lambda = 2$

6.4 For randomisation, the treatments and blocks are allocated numbers at random and then treatments are randomised within each block over the experimental units according to the plan of the design.

6.5 For the analysis of an experiment, the following are computed:

a) Correction factor (CF) = G^2/tr, where G is the grand total of the observations;

b) Total sum of squares (S_{tot}), given by squaring each value, adding them, and subtracting CF from it;

c) Treatment totals for all the t treatments, denoted by (say) T_1, T_2,...T_t. Since each treatment is replicated r times, each treatment total is based on r observations;

d) Block totals for all the b blocks, denoted by B_1, B_2,....B_b. Each block total is the sum of k observations, since each block contains k observations;

e) Adjusted treatment totals denoted by Q_1, Q_2 .... Q_t for all the treatments are given by subtracting from the treatment total the ratio of the block totals where the particular treatment occurs, to the number of units in each blocks,

$$Q_1 = T_1 - \frac{1}{k} \sum_{j(1)} B_j$$

$$Q_2 = T_2 - \frac{1}{k} \sum_{j(2)} B_j$$

.

.

.

$$Q_t = T_t - \frac{1}{k} \sum_{j(t)} B_j$$

In the above expressions, the sum $\sum_{j(i)} B_j$ denotes the sum of all those block totals which contain the i = th treatment. It may be verified that $\sum Q_i = 0$. For instance, in **6.3** the various adjusted totals are:

$$Q_1 = T_1 - \tfrac{1}{3} (B_1 + B_2 + B_3 + B_6 + B_{10})$$
$$Q_2 = T_2 - \tfrac{1}{3} (B_1 + B_2 + B_4 + B_7 + B_9)$$
$$Q_3 = T_3 - \tfrac{1}{3} (B_3 + B_4 + B_5 + B_6 + B_7)$$
$$Q_4 = T_4 - \tfrac{1}{3} (B_3 + B_4 + B_8 + B_9 + B_{10})$$
$$Q_5 = T_5 - \tfrac{1}{3} (B_1 + B_5 + B_7 + B_8 + B_{10})$$
$$Q_6 = T_6 - \tfrac{1}{3} (B_2 + B_5 + B_6 + B_8 + B_9)$$

Therefore $\sum Q_i = 0$

f) Efficiency factor (E) = $\lambda t/rk$

NOTE — The efficiency factor measures the efficiency of balanced incomplete block design over randomised block design.

g) Adjusted treatment sum of squares (S'_T) is given by:

$$S'_T = \frac{k}{\lambda t} (Q_1^2 + Q_2^2 \text{ ...... } + Q_t^2)$$

h) The sum of squares (unadjusted) for blocks is given by:

$$S_B = \frac{1}{k} (B_1^2 + B_2^2 + \text{......} + B_b^2) - CF$$

j) The sum of squares (unadjusted) for treatments is given by:

$$S_T = \frac{1}{r} (T_1^2 + T_2^2 + \text{......} + T_t^2) - CF$$

k) Sum of squares due to error (S_E) is obtained by subtracting sum of squares of adjusted treatments and unadjusted blocks from total sum of squares.

6.7 The different sums of squares can be entered in the analysis of variance table given in Table 13.

TABLE 13 ANALYSIS OF VARIANCE TABLE FOR BALANCED INCOMPLETE BLOCK DESIGN

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SQUARE	F-VALUE
(1)	(2)	(3)	(4)	(5)
Between blocks (unadjusted)	b-1	$S_B = \frac{1}{k} \sum_{i=1}^{b} B_i^2 - CF$	$MS_B = S_B/(b-1)$	
Between treatments (adjusted)	t-1	$S_T = \frac{1}{rE} \sum_{i=1}^{t} Q_i^2$	$MS_T = S_T/(t-1)$	MS_T/MS_E
Experimental error	tr-b-t + 1	$S_E = S_{tot} - S_B - S_T$	$MS_E = \frac{S_E}{(tr-b-t+1)}$	
Total	tr-1	$S_{tot} = \Sigma\Sigma\, y^2 - CF$		

6.8 For testing the differences among treatments, the mean square for treatments (adjusted) is compared with the error mean square, that is, compute $F = MS_T/MS_E$. If calculated value of F is greater than the tabulated value of F for [(t-1), (tr-b-t + 1)] degrees of freedom at desired level of significance, it is inferred that the treatment effects are not equal. In order to test whether the block effects are equal or not, the adjusted block sum of squares is calculated by the following identity :

Adjusted treatment sum of squares + Unadjusted block sum of squares

= Adjusted block sum of squares + Unadjusted treatment sum of squares

6.8.1 The adjusted block mean square is compared with the error mean square. The calculated value of F compared with corresponding tabulated value will indicate whether there is significant difference among the blocks or not.

6.9 Merits and Demerits

6.9.1 *Merits* — When the number of treatments are large, the experimental conditions may not permit blocks large enough to include every treatment, so that all the treatments cannot be tested under uniform conditions. In such cases the designs discussed earlier in this standard can not be applied and balanced incomplete blocks designs are used.

6.9.2 *Demerits*

a) The analysis of balanced incomplete block designs is more complicated as compared to other designs.

b) This design has many constraints, like tr = bk, λ (t-1) = r (k-1). Therefore theoretically it may be possible to construct BIBD for a given value of k and t, but most of these are of little use in practice because it will require large value of 'r'.

6.10 Example 4 — It was desired to make inter comparison of seven thermometers, denoted by A, B, C, D, E, F and G. Because of limitations of the testing equipments, these thermometers were tested in sets of three with the following arrangements.

(A, B, D), (E, F, A), (B, C, E), (F, G, B), (C, D, F), (G, A, C) and (D, E, G).

The thermometers had scale divisions of one-tenth of a degree and were read to the third place of decimal with an optical aid. The readings were made just above 30°C and for convenience only the last two places are entered, that is the entry 56 means 30.056. The data is recorded in Table 14.

TABLE 14 THERMOMETER READINGS

	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6	Set 7
	A 56	E 16	B 41	F 46	C 54	G 36	D 50
	E 31	F 41	C 53	G 32	D 43	A 68	E 32
	D 35	A 58	E 24	B 46	F 50	C 60	G 38
Total	122	115	118	124	147	164	120

6.10.1 The various calculations are as given below:

a) Grand total = 910

b) Thermometer totals:

A	B	C	D	E	F	G
182	118	167	128	72	137	106

c) Adjusted totals (Q_i) :

Q_A	Q_B	Q_C	Q_D	Q_E	Q_F	Q_G
48.33	– 3.33	24.00	– 1.66	– 45.67	8.33	– 30.00

d) Adjusted sum of squares for thermometers = $\frac{\sum_i Q_i^2}{rE}$

Here r = 3, $E = \frac{\lambda t}{rk} = \frac{1 \times 7}{3 \times 3}$ = 7/9; thus rE = 7/3.

Adjusted sum of squares for thermometers

$$= \frac{3}{7}(Q_A^2 + Q_B^2 + \ldots + Q_G^2)$$

$$= \frac{3}{7}(5\,980.771\,2) = 2\,563.19$$

e) Correction factor = $\frac{(910)^2}{21}$ = 39 433.33

f) Unadjusted sum of squares for sets

$$= \frac{1}{3}[(122)^2 + (115)^2 + \ldots + (120)^2] - CF$$

= 671.33

g) Total sum of squares = 42 698 – CF = 3 264.67

h) Error sum of squares = 3 264.67 – 2 563.19 – 671.33 = 30.15

j) Unadjusted sum of squares between thermometers

$$= \frac{1}{3}[(182)^2 + (118)^2 + \ldots (106)^2] - CF = 2\,736.67$$

k) Adjusted sum of squares for sets = 671.33 + 2 563.19 – 2 736.67 = 497.85

6.10.2 The analysis of variance table is given in Table 15.

6.10.3 The tabulated value of F for (6, 8) degrees of freedom and at 5 percent level is 3.58. Thus, the thermometers and sets can be regarded as significantly different.

7. FACTORIAL EXPERIMENTS

7.1 The testing of a number of treatments, not necessarily related to each other have been discussed earlier in randomised blocks, and latin square designs. However, in industrial applications, several factors may affect the characteristic under study and it is intended to estimate the effect of each of the factors and how the effect of one factor varies over the levels of the other factors. In such situation, the logical procedure would be to vary all factors simultaneously within the framework of the same experiment. Such experiments are known as factorial experiments.

TABLE 15 ANALYSIS OF VARIANCE TABLE

(*Clause* 6.10.2)

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F-value	Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F-value
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Between Thermometers (adjusted)	6	2 563.19	427.20	113.3	Thermometers (unadjusted)	6	2 736.67	—	
Between sets (unadjusted)	6	671.33	—		Sets (adjusted)	6	497.85	82.98	22.01
Experimental error	8	30.15	3.77		Experimental error	8	30.15	3.77	
Total	20	3 264.67			Total	20	3 264.67		

7.2 The factorial experiments are particularly useful in experimental situations which require the examination of the effects of varying two or more factors. In such situations, it is not sufficient to vary one factor at a time; all combinations of the different factor levels must be examined in order to elucidate the effect of each factor and the possible ways in which each factor may be modified by the variation of the others. In the analysis of the experimental results, the effect of each factor can be determined with the same accuracy as if only one factor had been varied at a time and the interaction effects between the factors can also be evaluated.

7.3 Designs with Factors at Two Levels (2^n Series) — The simplest class of factorial experiment is that involving factors at two levels, that is, the 2^n series, *n* being the number of factors examined in the experiment. The notations being used in the 2^n designs by Yates, and the calculations of main effects and the interactions are as given in **7.3.1** to **7.3.3**.

7.3.1 *Notation* — The letters A, B, C ... denote the factors and the levels of A, B, C ... are denoted by (1), a; (1), b; (1), c; ... respectively. As a convention, the lower case letters a, b, c ... denote the higher levels of the factors. The low level is signified by the absence of the corresponding letter. Thus the treatment combination bd, in a 2^4 factorial experiment, means the treatment combination which contains the first (low) level of factor A and C, and the second (high) level of factors B and D. The treatment combination which consists of the first level of all factors is denoted by the symbol (1). The letters A, B, C, AB, when they refer to numbers, represent respectively the main effects of factors A, B, C and the interaction effect of factors A and B respectively.

7.3.2 *Main Effects and Interactions* — The change in the average response produced by a change in the level of the factor is called its "main effect." It may so happen sometimes that the effect of one factor is different at different levels of one or more of the other factors, in this case the two factors are said to interact each other. The interaction between two factors is termed as "First order interaction", or "Two factor interaction" and is denoted by AB. If the interaction between two factors AB is different at different levels of a third factor C, then there is said to be an interaction between the three factors. This is referred to as 'second order interaction' or 'three factor interaction' and is denoted by ABC. Similarly the third and higher order interactions are defined.

7.3.3 Yates has developed a systematic tabular method for calculation of main effects and interaction for 2^n factorial experiment. The various steps in the computation are explained below with the help of Table 18.

a) Arrange the treatment combinations in standard order as shown in Column 1 titled 'treatment combinations'.

b) Place the sum of values of each of the treatment combinations (from Table 16) in the second column titled 'Total'.

c) Derive the top half of the third column titled 'I' by adding the values in pairs from the second column. The lower half of this column is obtained by taking the difference of same pairs, the first value of each pair being subtracted from the second value.

d) By the procedure given in (c), columns 4, 5 and 6 titled as II, III and IV are obtained from the columns 3, 4 and 5 respectively.

e) Repeat the procedure *n* times, where *n* is the number of factors involved in the experiment. For Table 18, the procedure has been repeated 4 times thereby obtaining columns titled I to IV.

f) Obtain the mean factorial effect by dividing the total factorial effect by $r.2^{n-1}$ where r is the number of replicates. Column 8 is obtained by dividing each value of column 6 by 16 ($= 2 \times 2^3$).

g) The sum of squares due to different treatment combinations of the factorial effects (main effects and inter-actions) are obtained by dividing the squares of the factorial effects total by $r.2^n$. Column 9 is obtained by dividing the square of each value of column 6 by 32 ($= 2 \times 2^4$). The mean squares (*see* column 10) is same as sum of squares, as the degrees of freedom for each factorial effect is one.

7.4 Merits and Demerits

7.4.1 *Merits*

a) This design makes maximum utilization of all results and every result is used to evaluate each factor,

b) It can measure interaction of factors,

c) The experimental error tends to be lower than other designs, and

d) The final calculations have broader applicability because of scope of experimental trials.

7.4.2 *Demerits*

a) The experiment can be too large when all combinations of factors and levels are run, and

b) The size of the experiment requires a larger amount of homogeneous material than other designs.

7.5 Example — In an experiment, four factors A, B, C, and D each at two levels are studied. The response obtained for each treatment combinations corresponding to two replicates is given in Table 16. Determine with the help of a factorial experiment as to which of the main effects and interactions are significant.

The various calculations are as given below :

a) Correction factor $= \frac{(2\ 542.5)^2}{32} = 202\ 009.6$

TABLE 16 VALUES FOR 2^4 FACTORIAL EXPERIMENT

(*Clause* 7.5)

FACTOR A	FACTOR B	FACTOR C: I level, Factor D: I level	FACTOR C: I level, Factor D: II level	FACTOR C: II level, Factor D: I level	FACTOR C: II level, Factor D: II level
I level	I level	27.3, 24.7 (1)	26.2, 23.9 d	58.7, 43.4 c	50.1, 49.5 cd
	II level	86.3, 93.9 b	98.2, 92.4 bd	80.2, 69.3 bc	92.0, 86.7 bcd
II level	I level	79.6, 75.5 a	76.5, 72.9 ad	101.8, 105.8 ac	78.4, 74.3 acd
	II level	125.8, 97.8 ab	130.7, 134.5 abd	82.1, 87.4 abc	110.5, 106.1 abcd

b) Total sum of squares = $\sum_i \sum_j y_{ij}^2$ – CF = 27 391.1

c) Sum of squares due to replicates

$$= \frac{(1\ 304.4)^2}{16} + \frac{(1\ 238.1)^2}{16} - CF = 137.4$$

d) Sum of squares due to treatments = 26 694.8

e) Sum of squares due to experimental error
= 27 391.1 – 137.4 – 26 694.8 = 558.9

7.5.1 The above sum of squares may be entered in the analysis of variance table as given in Table 17.

7.5.2 The tabulated value of F for (15, 15) degrees of freedom and 5 percent level of significance is 2.40. Since the calculated value of F is greater than tabulated value, the treatment effect is significant.

7.5.3 Since the treatment effect is significant, the next step is to determine as to which of the main effects and interactions are significant. For this purpose, the mean squares of main effects and interactions (*see* col 10 of Table 18) are compared with error mean square. The calculated values of F so obtained (*see* col 11 of Table 18) are then compared with, 4.54 the tabulated value of F for (1,15) degrees of freedom and at 5 percent level of significance. From the last column of Table 18, it is inferred that the main effects A and B, the two factor interactions AB, BC, AC and BD; and three factor interactions ABD and BCD are significant.

TABLE 17 ANALYSIS OF VARIANCE TABLE FOR 2^4 DESIGN

(*Clause* 7.5.1)

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SQUARE	F-VALUE
(1)	(2)	(3)	(4)	(5)
Treatments	15	26 694.8	1 779.7	47.71
Replications	1	137.4	137.4	3.68
Experimental error	15	558.9	37.3	
Total	31	27 391.1		

TABLE 18 YATES' METHOD FOR 2^4 EXPERIMENT

(*Clauses* 7.3.3 *and* 7.5.3)

Treatment Combination	Total	I	II	III	IV	Effect	Mean Effect	S.S.	M.S.	F
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
(1)	52.0	207.1	610.9	1 239.6	2 542.6	G				
a	155.1	403.8	628.7	1 302.9	536.9	A	33.6	9 008.2	9 008.2	241.5*
b	180.2	309.7	655.3	272.0	605.3	B	37.8	11 449.9	11 449.9	307.0*
ab	223.6	319.0	647.6	264.9	–185.1	AB	–11.6	1 070.7	1 070.7	28.7*
c	102.1	199.5	146.5	206.0	10.1	C	0.6	3.2	3.2	0.1
ac	207.6	455.8	125.5	399.3	–103.9	AC	–6.5	337.4	337.4	9.0*
bc	149.5	252.3	173.9	–145.2	–300.7	BC	–18.8	2 825.6	2 825.6	75.8*
abc	169.5	395.3	91.0	–39.9	–16.3	ABC	–1.0	8.3	8.3	0.2
d	50.1	103.1	196.7	17.8	63.3	D	4.0	125.2	125.2	3.4
ad	149.4	43.4	9.3	–7.7	–7.1	AD	–0.4	1.6	1.6	0.03
bd	190.6	105.5	256.3	–21.0	193.3	BD	12.1	1 167.7	1 167.7	31.3*
abd	265.2	20.0	143.0	–82.9	105.3	ABD	6.6	346.5	346.5	9.3*
cd	99.6	99.3	–59.7	–187.4	–25.5	CD	–1.6	20.3	20.3	0.5
acd	152.7	74.6	–85.5	–113.3	–61.9	ACD	–3.9	119.7	119.7	3.2
bcd	178.7	53.1	–24.7	–25.8	74.1	BCD	4.6	171.6	171.6	4.6*
abcd	216.6	37.9	–15.2	9.5	35.3	ABCD	2.2	38.9	38.9	1.0
Total	2 542.5							26 694.8		

*These are significant at 5 percent level of significance. For further details, refer to **7.5.3**.

APPENDIX A

(*Clause* 5.3)

LIST OF STANDARD LATIN SQUARES

4 × 4

I		II	
	A B C D		A B C D
	B A D C		B A D C
	C D B A		C D A B
	D C A B		D C B A
	1-3		4

5 × 5

I		II		III	
	A B C D E		A B C D E		A B C D E
	B A E C D		B A D E C		B C E A D
	C D A E B		C E A B D		C E D B A
	D E B A C		D C E A B		D A B E C
	E C D B A		E D B C A		E D A C B
	1-25		26-50		51-56

6 × 6

I		II		III		IV	
	A B C D E F		A B C D E F		A B C D E F		A B C D E F
	B C F A D E		B C F E A D		B C F E A D		B A F E C D
	C F B E A D		C F B A D E		C F B A D E		C F B A D E
	D E A B F C		D A E B F C		D E A B F C		D C E B F A
	E A D F C B		E D A F C B		E A D F C B		E D A F B C
	F D E C B A		F E D C B A		F D E C B A		F E D C A B
	0 001-1 080		1 081-2 160		2 161-3 240		3 241-4 320

V		VI		VII		VIII	
	A B C D E F		A B C D E F		A B C D E F		A B C D E F
	B A E F C D		B A E C F D		B A F E D C		B A F E C D
	C F B A D E		C F B A D E		C E B F A D		C F B A D E
	D E A B F C		D E F B C A		D C A B F E		D E A B F C
	E D F C B A		E D A F B C		E F D C B A		E C D F B A
	F C D E A B		F C D E A B		F D E A C B		F D E C A B
	4 321-5 400		5 401-5 940		5 941-6 480		6 481-7 020

IX	A	B	C	D	E	F
	B	C	D	E	F	A
	C	E	A	F	B	D
	D	F	B	A	C	E
	E	D	F	B	A	C
	F	A	E	C	D	B
	7 021- 7 560					

X	A	B	C	D	E	F
	B	A	E	F	C	D
	C	F	A	E	D	B
	D	C	B	A	F	E
	E	D	F	C	B	A
	F	E	D	B	A	C
	7 561-7 920					

XI	A	B	C	D	E	F
	B	A	F	C	D	E
	C	E	A	B	F	D
	D	F	E	A	C	B
	E	C	D	F	B	A
	F	D	B	E	A	C
	7 921-8 280					

XII	A	B	C	D	E	F
	B	A	E	F	C	D
	C	F	A	B	D	E
	D	E	B	A	F	C
	E	D	F	C	B	A
	F	C	D	E	A	B
	8 281-8 640					

XIII	A	B	C	D	E	F
	B	C	F	A	D	E
	C	F	B	E	A	D
	D	A	E	B	F	C
	E	D	A	F	C	B
	F	E	D	C	B	A
	8 641-8 820					

XIV	A	B	C	D	E	F
	B	C	A	F	D	E
	C	A	B	E	F	D
	D	F	E	B	A	C
	E	D	F	C	B	A
	F	E	D	A	C	B
	8 821-8 940					

XV	A	B	C	D	E	F
	B	C	A	F	D	E
	C	A	B	E	F	D
	D	F	E	B	C	A
	E	D	F	A	B	C
	F	E	D	C	A	B
	8 941-9 060					

XVI	A	B	C	D	E	F
	B	C	A	E	F	D
	C	A	B	F	D	E
	D	E	F	B	A	C
	E	F	D	A	C	B
	F	D	E	C	B	A
	9 061-9 180					

XVII	A	B	C	D	E	F
	B	C	A	F	D	E
	C	A	B	E	F	D
	D	F	E	B	A	C
	E	D	F	A	C	B
	F	E	D	C	B	A
	9 181-9 240					

XVIII	A	B	C	D	E	F
	B	C	A	E	F	D
	C	A	B	F	D	E
	D	F	E	B	A	C
	E	D	F	C	B	A
	F	E	D	A	C	B
	9 241-9 280					

XIX	A	B	C	D	E	F
	B	A	F	E	D	C
	C	D	A	B	F	E
	D	F	E	A	C	B
	E	C	B	F	A	D
	F	E	D	C	B	A
	9 281-9 316					

XX	A	B	C	D	E	F
	B	A	D	F	C	E
	C	F	A	E	B	D
	D	E	B	A	F	C
	E	D	F	C	A	B
	F	C	E	B	D	A
	9 317-9 352					

XXI	A	B	C	D	E	F
	B	A	E	C	F	D
	C	E	A	F	D	E
	D	C	F	A	B	E
	E	F	D	B	A	C
	F	D	B	E	C	A
	9 353-9 388					

XXII	A	B	C	D	E	F
	B	C	A	F	D	E
	C	A	B	E	F	D
	D	E	F	A	B	C
	E	F	D	C	A	B
	F	D	E	B	C	A
	9 389-9 408					

7 × 7
A B C D E F G
B D E F A G C
C G F E B A D
D E A B G C F
E C B G F D A
F A G C D E B
G F D A C B E

8 × 8
A B C D E F G H
B C A E F D H G
C A D G H E F B
D F G C A H B E
E H B F G C A D
F D H A B G E C
G E F H C B D A
H G E B D A C F

9 × 9
A B C D E F G H I
B C E G D I F A H
C D F A H G I E B
D H A B F E C I G
E G B I C H D F A
F I H E B D A G C
G F I C A B H D E
H E G F I A B C D
I A D H G C E B F

10 × 10
A B C D E F G H I J
B G A E H C F I J D
C H J G F B E A D I
D A G I J E C B F H
E F H J I G A D B C
F E B C D I J G H A
G I F B A D H J C E
H C I F G J D E A B
I J D A C H B F E G
J D E H B A I C G F

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